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Retarded Green Functions and Modified Dispersion Relations

Daniel Arteaga,[1,4] Renaud Parentani,[2] and Enric Verdaguer[3]

We analyse a three-field model, which describes a relativistic two-level atom interacting with a radiation bath. From the one-loop retarded propagators at finite temperature we extract the transition rates and the modifications of the dispersion relations. To further investigate the relationships between propagators and these physical quantities, we analyse a non-equilibrium situation in which an additional atom is present in the bath. Preliminary results indicate that transition rates can still be extracted from the (retarded) propagator. This approach could therefore be useful in relating high-frequency (trans-Planckian) dispersion relations to the physical processes occurring at these scales.

KEY WORDS: dispersion relation; decay rate; thermal field theory; non-equilibrium field theory.

1. INTRODUCTION AND MOTIVATION

In quantum field theory, for stable particles in the Minkowski vacuum, the energy as a function of the momentum—i.e., the dispersion relation—is determined by the position of the poles of the two-point Feynman propagator in the complex energy plane. The poles always lie on the real axis in the case of stable particles. This relation between the pole structure and the energy is valid even non-perturbatively (Peskin and Schroeder, 1998; Weinberg, 1995). Since the particle propagation is Lorentz invariant, the dispersion relation $E^2 = m^2 + \mathbf{p}^2$ and, in general, the details of the particle propagation are fully determined by one parameter, the physical mass of the particle m.

[1] Departament de Física Fonamental, Universitat de Barcelona, Av. Diagonal 647, 08028 Barcelona, Spain.
[2] Laboratoire de Physique Théorique, CNRS UMR 8627, Université Paris XI, 91405 Orsay Cedex, France.
[3] Departament de Física Fonamental and CER en Astrofísica, Física de Partícules i Cosmologia, Universitat de Barcelona, Av. Diagonal 647, 08028 Barcelona, Spain.
[4] To whom correspondence should be addressed at Departament de Física Fonamental, Universitat de Barcelona, Av. Diagonal 647, 08028 Barcelona, Spain; e-mail: darteaga@ub.edu.

When the particles are unstable, asymptotic regions do no longer exist in a strict sense and hence the particle concept is also an approximation. The pole of the propagator acquires an imaginary part, which is proportional to the decay rate of the unstable particle, according to the optical theorem (Peskin and Schroeder, 1998; Weinberg, 1995). The real part of the pole still gives the energy of the particle as a function of the momentum, but now only in an approximate way, in accordance with the energy–time uncertainty principle.

The situation gets more involved when the Minkowski vacuum is replaced by a less trivial state such as a thermal background (Das, 1997; le Bellac, 1996; Landsman and van Weert, 1987) or a curved spacetime background (Birrell and Davies, 1982; Wald, 1994). Several comments are in order. First, in these backgrounds, the closed time path (CTP) formalism of field theory has to be applied (Chou *et al.*, 1985; Keldysh, 1965; Schwinger, 1961). Second, it remains to be seen whether the real and imaginary parts of the pole of some propagator still admit similar interpretations as dispersion relations and decay rates. Third, even if such an interpretation is found, it must be noted that the interactions with the background excitations will induce terms in the self-energy which will effectively break the (local) Lorentz invariance of the free propagation. Last, one expects a richer phenomenology (including for instance dissipation, dispersion and decoherence), which cannot be probably fully extracted from a two-point propagator. Our endeavour is to determine the set of physical observables that can be extracted from the two-point propagator.

In the case of a thermal background, Weldon (1983) showed that the imaginary part of the poles of the retarded propagator admitted an interpretation, not as a decay rate, but rather as a rate of approach to thermal equilibrium. Although he did the demonstration at one loop, it can be generalized to any order in perturbation theory (Bedaque *et al.*, 1997). Concerning the real part of the poles, it is usually assumed that it still indicates the energy of the particle as a function of the momentum. Although there are several arguments in support of this interpretation (see for instance Arteaga *et al.*, 2004a; Donoghue *et al.*, 1985), we are not aware of a formal proof.

A rather different description of the particle propagation in a thermal bath is the one provided by the open quantum system approach, working in the formalism of first quantization. There one assumes that a full quantum system can be divided in two different parts, the distinguished subsystem, consisting in our case of the particle in which we are interested, and the environment, consisting in the thermal bath. The particle dynamics can be described by the time evolution of the reduced density matrix $\rho_{\mathrm{r}}(\mathbf{x}, \mathbf{x}'; t)$, or equivalently by the corresponding Wigner function $W_{\mathrm{r}}(\mathbf{x}, \mathbf{p}; t)$ (Hillery *et al.*, 1984). The equation describing the time evolution is called the master equation; and from it all the phenomena associated to the propagation of the particle can be extracted in principle, even in non-equilibrium situations.

One can find a link between the two different descriptions, at least in the case in which the particle field is sufficiently massive so as to remain thermally unexcited. In this case, the distinguished system is a single excitation of the particle field and the environment is the radiation field. The reduced density matrix for the one-particle state $\hat{\rho}_{\mathrm{r}}$ can be computed from the state of the quantum fields $\hat{\rho}$ by considering just the one-particle sector of the particle field and tracing over the environment degrees of freedom. Schematically,

$$\hat{\rho}_{\mathrm{r}} = \mathrm{Tr}_{\mathrm{env}}(\hat{P}\hat{\rho}\hat{P}), \tag{1}$$

where $\hat{P}$ is the operator that selects the one-particle sector from the field density matrix. The state of the fields $\hat{\rho}$ will initially correspond to a one-particle excitation on top of a thermal radiation field. In this situation, the dynamics of the reduced density matrix encodes most of the information about the particle propagation.[5] However, as we have commented, some of the relevant aspects of the particle propagation may also probably be deduced from the analytic structure of the retarded equilibrium propagator.

In the case of the propagation in a curved spacetime background one expects a similar situation. Again, the CTP formalism has to be employed. Interaction with the virtual radiation modes in the curved background will induce qualitatively similar phenomena in the propagating particles, such as particle decay, dispersion or modification of the dispersion relation. In particular, one expects that the effects associated with the radiative gravitational corrections in a non-trivial background may significantly modify the particle dynamics when the Planck scale is reached. They should therefore play an important role in the resolution of the trans-Planckian question (Jacobson, 1991), both in black hole physics (Brout *et al.*, 1995; Helfer, 2003) and cosmology (Martin and Brandenberger, 2001; Niemeyer and Parentani, 2001). By analogy to the thermal case, we expect that at least some of the relevant aspects of the particle propagation can be deduced from the analytic structure of the propagator, see Arteaga *et al.* (2004a).

Since the propagation in a curved spacetime background closely resembles, from a qualitative point of view, the propagation in a thermal background, in Arteaga *et al.* (2004b) we studied the propagation of a scalar particle in a thermal bath of gravitons, in order to gain some physical insight in the corresponding curved background situation. We computed the one-loop correction to the position of the pole of the retarded propagator. Unfortunately, us this correction was purely real at one loop, due to kinematica constraints in the relevant one-loop diagrams. This prevented us from analysing properly the dissipative effects associated with the imaginary part. However, there is no reason to expect that the imaginary part

[5] Note, however, that not all the information is encoded in the reduced density matrix: some quantities involving evaluation at different times cannot be extracted from the time evolution of the reduced density matrix (Calzetta *et al.*, 2003).

will vanish at higher orders in perturbation theory, neither in the thermal case nor in curved backgrounds.

In this paper, we analyse a simpler physical system that is free of the kinematical constraints that made the imaginary part of the pole vanish, but nevertheless shows the same kind of dissipative behaviour we could expect in the gravitational interaction at higher loops. The system contains two massive fields, which interact with a massless radiation field. We first analyse an equilibrium situation: namely the propagation in a thermal background. We then analyse the same system in a different background, consisting of a thermal bath of gravitons plus a one-particle excitation. There are several motivations for studying this second case. First, we analyse the system in a background which is not thermal and thus is not tied by the particular, non-generic, properties of thermal field theory. Second, the intermediate states of a particle decay will naturally involve non-equilibrium backgrounds characterized by the presence of additional one-particle states. Third, the computation of the reduced density matrix from a field theory setting, following Eq. (1), will probably also involve backgrounds with one-particle states. Finally, we will check whether the interpretation of the imaginary part of the poles of the retarded propagator as rates of approach to thermal equilibrium still holds in this particular non-equilibrium situation.

The plan of the paper is as follows: In Section 2 we introduce the model. In Sections 3 and 4 we present the results of the one-loop correction to the propagator and discuss its interpretation, both in the vacuum (Section 3) and in a thermal bath (Section 4). In Section 4 we also stress the need of the CTP approach in backgrounds different than the vacuum. With similar techniques as in Section 4, in Section 5 we discuss the corrections to the propagator in a non-equilibrium background characterized by the presence of an additional atom. Finally, in Section 6 we summarize and discuss the main results.

We use a system of units with $\hbar = c = k_{\mathrm{B}} = 1$. The signature of the metric will be $(-, +, +, +)$.

2. A THREE-FIELD MODEL

We consider a quantum field model consisting of two massive scalar fields ϕ_M and ϕ_m, of masses M and m, respectively, and a massless scalar field χ, which interact via a trilinear interaction, with a coupling constant $\tilde{g}$. The action for the model is:

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu\phi_M\partial^\mu\phi_M + M^2\phi_M^2 + \partial_\mu\phi_m\partial^\mu\phi_m + m^2\phi_m^2 + \partial_\mu\chi\,\partial^\mu\chi + 2\tilde{g}\phi_M\phi_m\chi\right), \tag{2}$$

where we consider that $M > m$. Notice that in this model the coupling constant $\tilde{g}$ has dimensions of energy. We prefer to work with the dimensionless coupling constant $g = \tilde{g}/m$.

The model allows for two equivalent alternative interpretations: the excitations of the two fields ϕ_M and ϕ_m may correspond to two different particles, or either they may correspond to two different internal degrees of freedom of the same particle. In other words, the doublet (ϕ_m, ϕ_M) may represent a particle with two internal states, one being more energetic than the other.

We will be interested in considering the limit in which the masses of the two fields are very large, while their mass difference $\Delta m = M - m$ is of the order of magnitude E of the energies involved in the problem: $M, m \gg \Delta m$ and $\Delta m \sim E$. In this limit, the system can be considered a relativistic two-level atom of mass m, represented by (ϕ_M, ϕ_m), interacting with a radiation field. This model was already used in Parentani (1995) to study the consequences of the recoils of a uniformly accelerated two-level atom subject to the Unruh (1976) effect.

From now on we will alternate the field-theoretic and atomic-like description and notations, often naming the particle represented by the doublet (ϕ_m, ϕ_M) as the "atom" and indicating quantities referring to the excited state ϕ_M with a star and quantities related to the fundamental state ϕ_m without the star.

3. GREEN FUNCTIONS IN THE VACUUM

At zero temperature, the self-energy $\Sigma^{(T=0)}(p^2)$ of the field ϕ_m is defined through

$$G_{\mathrm{F}}^{(T=0)}(p) = \frac{-i}{p^2 + m^2 + \Sigma^{(T=0)}(p^2)}, \tag{3}$$

where $G_{\mathrm{F}}^{(T=0)}(p)$ is the zero temperature Feynman propagator of ϕ_m. We recall that the self-energy can be computed as the sum of all one-particle irreducible diagrams with amputated external legs (see e.g. Peskin and Schroeder, 1998; Weinberg, 1995). At one loop, the self-energy corresponds to the diagrams in the Fig. 1 and for the case of the fundamental atomic state, the energy is found to be:

$$\Sigma^{(T=0)}(p^2) = \frac{g^2 m^2}{(4\pi)^2}\left[\frac{M^2}{p^2}\ln\left(1 + \frac{p^2}{M^2} - i\epsilon\right)\right.$$
$$\left. - \frac{M^2}{m^2}\ln\left(1 - \frac{m^2}{M^2}\right) + \ln\left(\frac{p^2 + M^2}{M^2 - m^2} - i\epsilon\right)\right]. \tag{4}$$

The calculation is standard and the details closely follow those of Arteaga *et al.* (2004b). For the excited state ϕ_M, the self-energy is found just by exchanging M with m: $\Sigma^{(T=0)*}(p^2) = \Sigma^{(T=0)}(p^2)[M \leftrightarrow m]$.

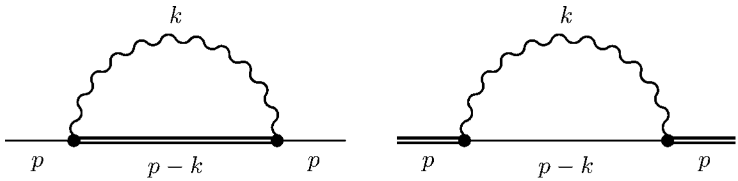

Fig. 1. Feynman diagrams leading to the one-loop self-energy of the fields ϕ_m and ϕ_M (respectively the fundamental and excited atomic states). Single straight lines represent the fundamental state, double lines the excited state, and curly lines the massless particle.

We have employed an on-shell renormalization scheme in order to absorb the divergences, so in this case the real part of the self-energy is zero at the mass-shell: Re $\Sigma^{(T=0)}(-m^2) = 0$. This ensures, according to the Källen–Lehman representation of the propagator, that the masses m and M correspond to the true masses of the two states of the atom.[6]

The decay rate, defined as the probability of decay per unit time, is proportional to the imaginary part of the self-energy at the mass-shell, according to the optical theorem (Peskin and Schroeder, 1998; Weinberg, 1995). For the excited stated the decay rate Γ_-^* is proportional to the mass gap:

$$\Gamma_-^* = -\frac{1}{M}\,\mathrm{Im}\,\Sigma^{T=0^*}(-M^2) \approx \frac{g^2}{8\pi}\Delta m, \tag{5a}$$

while the low-mass state is of course stable:

$$\Gamma_- = -\frac{1}{m}\,\mathrm{Im}\,\Sigma^{(T=0)}(-m^2) = 0. \tag{5b}$$

Notice that the decay rate of the heavy state, as expected, is governed by the mass gap and does not depend on the mass of the atom.

A similar analysis can be made for the self-energy Π of radiation field χ. Since there is no gauge invariance protecting the mass of the radiation field, the renormalization process will generate a mass term for the radiation particle. We will put to zero the renormalized mass, so that the real part of the self-energy vanishes. The imaginary part of the self-energy automatically vanishes on-shell, since a real massless particle cannot decay. However, the imaginary part of the self-energy is non-zero off-shell because the virtual excitations of the radiation field decay with a rate given by

$$\gamma_-(q^0, \mathbf{q}) = -\frac{1}{q^0}\,\mathrm{Im}\,\Pi^{(T=0)}(q^2) \tag{6}$$

in the laboratory rest frame.

[6] Strictly speaking, this is only true for the stable states such as ϕ_m, which are the only ones that have well-defined asymptotic modes.

4. GREEN FUNCTIONS IN A THERMAL BACKGROUND

In this section we will consider the situation in which the radiation field is not in its vacuum state but is in a thermally excited state corresponding to a temperature of the order of the mass gap Δm. This means that the massive scalar fields will remain unexcited in the vacuum.

We will incorporate the thermal effects into the self-energy through the real-time description of thermal field theory (Das, 1997; le Bellac, 1996; Landsman and Weert, 1987), which can be seen as a particular application of the CTP method in non-equilibrium field theory.

In the CTP approach, which can be applied for an arbitrary initial state $\hat{\rho}$, the number of degrees of freedom is doubled. One has to consider four propagators organized in a 2×2 matrix $G_{ab}(x, x')$. The 11 and 22 components correspond respectively to the Feynman (time-ordered) and Dyson (anti-time-ordered) propagators, and the off-diagonal components correspond to the Whightman functions (non-ordered). The self-energy also becomes a matrix $\Sigma^{ab}(x, x')$, defined through

$$G_{ab}(x, x') = G^{(0)}_{ab}(x, x') + \int d^4y\, d^4z\, G^{(0)}_{ac}(x, y) \left[-i\,\Sigma^{cd}(y, z)\right] G_{db}(z, x'), \quad (7)$$

where $G^{(0)}_{ab}(x, x')$ are the propagators of the free theory. Perturbation theory can be organized as usual, but taking into account that there will be two kinds of vertices and four kinds of propagators. In particular, the self-energy components $\Sigma^{ab}(x, y)$ correspond to the sum of all one-particle irreducible diagrams connecting an a with a b vertex.

An interesting combination is the retarded propagator $G_R(x, x') = G_{11}(x, x') - G_{12}(x, x')$, which is directly related to the so-called retarded self-energy $\Sigma_R(x, x') = \Sigma^{11}(x, x') + \Sigma^{12}(x, x')$ through

$$G_R(x, x') = G^{(0)}_R(x, x') + \int d^4y\, d^4z\, G^{(0)}_R(x, y)\left[-i\,\Sigma_R(y, z)\right] G_R(z, x'). \quad (8)$$

In general, the retarded and advanced propagators are the unique single combinations that verify a self-referent equation such as the previous one. However, notice that the computation of Σ_R requires the use of all the CTP propagators as explained earlier and hence it is not possible to do a perturbative expansion just in terms of the retarded propagator. In the thermal case, the state is homogeneous and stationary, so that the Fourier transform with respect to $x - x'$ can be introduced. In this case, the previous equation can be solved for $G_R(p)$:

$$G_R(p) = \frac{-i}{p^2 + m^2 + \Sigma_R(p)}. \quad (9)$$

In the Fourier representation, the retarded propagator is analytic in the upper p^0-plane.

As commented in Section 1, at finite temperature the real and imaginary parts of the poles of the retarded propagator, given by the on-shell values of the retarded self-energy, admit interpretations as the dispersion relations and decay rates, respectively. Although we will also compute the real part of the self-energy, in this paper we will mainly concentrate on the analysis of the imaginary part.

The imaginary part of the retarded self-energy can be expanded in general as

$$\mathrm{Im}\,\Sigma_{\mathrm{R}}(E_{\mathbf{p}}, \mathbf{p}) = -E_{\mathbf{p}}[\Gamma_{-}(\mathbf{p}) - \Gamma_{+}(\mathbf{p})], \tag{10}$$

where $\Gamma_{-}(\mathbf{p})$ and $\Gamma_{+}(\mathbf{p})$ are respectively the net decay and creation rates, which depend on the temperature T, and where $E_{\mathbf{p}}$ is the energy of the excitation. The decay rate $\Gamma_{-}(\mathbf{p})$ is the probability per unit time for an incoming particle with momentum $\mathbf{p}$ to decay to any other state into the thermal bath (this includes simply going to a different momentum state). Similarly, the creation rate $\Gamma_{+}(\mathbf{p})$ is the probability per unit time that the state of momentum $\mathbf{p}$ is spontaneously created from the thermal bath. The decay and creation rates verify the detailed equilibrium condition $\Gamma_{+}(\mathbf{p}) = e^{-E_{\mathbf{p}}/T}\,\Gamma_{-}(\mathbf{p})$. Differently to the vacuum case, they are usually computed in the laboratory reference system.

For an ensemble of particles obeying the Bose–Einstein statistics and described by the distribution function $f(\mathbf{p}, t)$, the transitions rates will be proportional to the actual value of the distribution. Similarly, the rate of increase will be proportional to the creation rate and, because of the Bose–Einstein statistics, will be favoured for those states already populated. These heuristic arguments suggest that the time evolution of its distribution function $f(\mathbf{p}, t)$, slightly departed from thermal equilibrium, is given by Weldon (1983)

$$\frac{\partial f(\mathbf{p}, t)}{\partial t} = -\Gamma_{-}(\mathbf{p}) f(\mathbf{p}, t) + \Gamma_{+}(\mathbf{p})[1 + f(\mathbf{p}, t)] \tag{11}$$

where the rates Γ have been computed with equilibrium distributions. Therefore this equation is only valid for distribution functions $f(\mathbf{p}, t)$ not far from thermal equilibrium; if the deviation from thermal equilibrium is larger, the decay and creation rates have to be computed with the out-of-equilibrium distributions, as we will see in the next section. Solving the previous differential equation we get

$$f(\mathbf{p}, t) = \frac{1}{e^{E_{\mathbf{p}}/T} - 1} - \Delta f_0(\mathbf{p})\, e^{-\Gamma(\mathbf{p})}, \tag{12}$$

where $\Delta f_0(\mathbf{p})$ is the initial departure from equilibrium and

$$\Gamma(\mathbf{p}) = \Gamma_{-}(\mathbf{p}) - \Gamma_{+}(\mathbf{p}) \tag{13}$$

is the rate of approach to the equilibrium. Comparing Eq. (10) with Eq. (13) we see that the imaginary part of the self-energy is proportional to the rate of approach to the equilibrium.

4.1. Atomic Fields ϕ_m and ϕ_M

In the case considered here the temperature T is much lower than the atom mass m, so that there are no thermal atoms in the bath. Since an atom cannot be spontaneously created from a thermal radiation field, $\Gamma_+(\mathbf{p}) = \Gamma_+^*(\mathbf{p}) = 0$ and the thermalization rate for the atomic states, $\Gamma(\mathbf{p})$ and $\Gamma^*(\mathbf{p})$, coincides with the net decay rate $\Gamma_-(\mathbf{p})$ and $\Gamma_-^*(\mathbf{p})$. Hence, Eq. (11) will not be of much use here (however, it will be useful in the next section). In terms of the decay amplitude $\mathcal{M}_-^*$ and $\mathcal{M}_-$, to leading order in the coupling constant the decay rates can be computed as

$$\Gamma_-^*(\mathbf{p}) = \frac{1}{2E_{\mathbf{p}}^*}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1+n(|\mathbf{q}|)}{2|\mathbf{q}|}\int \frac{d^3\mathbf{p}'}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}'}}|\mathcal{M}_-^*|^2 \times (2\pi)^4\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{p}')\delta(E_{\mathbf{p}}^* - |\mathbf{q}| - E_{\mathbf{p}'}), \tag{14a}$$

$$\Gamma_-(\mathbf{p}) = \frac{1}{2E_{\mathbf{p}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{n(|\mathbf{q}|)}{2|\mathbf{q}|}\int \frac{d^3\mathbf{p}'}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}'}^*}|\mathcal{M}_-|^2 \times (2\pi)^4\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{p}')\delta(E_{\mathbf{p}} + |\mathbf{q}| - E_{\mathbf{p}'}^*), \tag{14b}$$

where $E_{\mathbf{p}} = \sqrt{m^2+\mathbf{p}^2}$ and $E_{\mathbf{p}}^* = \sqrt{M^2+\mathbf{p}^2}$, and where $n(E)$ is the Bose–Einstein distribution function,

$$n(E) = \frac{1}{e^{E/T}-1}. \tag{15}$$

These two processes correspond to the Feynman diagrams shown in Fig. 2. The statistical factors $n(|\mathbf{q}|)$ and $1+n(|\mathbf{q}|)$ take into account the presence of radiation in the bath. The squared decay amplitudes simply correspond to $|\mathcal{M}_-|^2 = |\mathcal{M}_-^*|^2 = \tilde{g}^2 = m^2g^2$.

Fig. 2. Feynman diagrams giving the decay rate of the excited and fundamental atom states with momentum **p** respectively. On the left, the excited state decays to the fundamental state by emitting a massless particle into the thermal bath. On the right, the fundamental state absorbs a massless particle from the bath and gets excited.

The real and imaginary parts of the retarded self-energy can be obtained from the various self-energy components. The one-loop ϕ_m self-energy is, see Fig. 1,

$$-i\Sigma^{ab}(p) = -g^2 m^2 \int \frac{d^4k}{(2\pi)^4} G^{(0)*}_{ab}(p-k)\Delta^{(0)}_{ab}(k), \tag{16}$$

where $G^{(0)*}_{ab}(p)$ is the free vacuum propagator for the massive field ϕ_M, $\Delta^{(0)}_{ab}(p)$ is the free thermal propagator for the radiation field, and no implicit summation is assumed in this equation. Since the details of the calculation are very similar to those of Arteaga *et al.* (2004b), here we will just present the results.

Concerning the real part, the one-loop contribution is the following:

$$\text{Re}\,\Sigma_{\text{R}}(E_{\mathbf{p}}, \mathbf{p}) \approx -\frac{g^2}{6}\frac{T^2 m}{\Delta m}, \qquad \text{Re}\,\Sigma^*_{\text{R}}(E^*_{\mathbf{p}}, \mathbf{p}) \approx \frac{g^2}{6}\frac{T^2 m}{\Delta m}, \tag{17}$$

The fact that both contribution have opposite sign is simply due to the fact that both contributions are related by the exchange $m \leftrightarrow M$. Notice that the result is momentum independent; similar to this happens in electrodynamics (Donoghue *et al.*, 1985) and gravitation (Arteaga *et al.*, 2004b). Hence, the real part of the self-energy can be absorbed in the thermal mass

$$m_T^2 \approx m^2 - \frac{g^2}{6}\frac{T^2 m}{\Delta m}, \qquad M_T^2 \approx M^2 + \frac{g^2}{6}\frac{T^2 m}{\Delta m}. \tag{18}$$

The effective dispersion relations $E^2 = m_T^2 + \mathbf{p}^2$ and $E^{*2} = M_T^2 + \mathbf{p}^2$ are thus Lorentz-invariant. This is not what one would expect *a priori*, since the thermal bath introduces a privileged reference frame and there is no immediate reason why the effective dispersion relation should preserve the frame independence.

Concerning the imaginary part, for the sake of simplicity we just quote the results in the non-relativistic limit, $|\mathbf{p}| \ll m$. For the excited state we have

$$\begin{aligned}\text{Im}\,\Sigma^*_{\text{R}}(E^*_{\mathbf{p}}, \mathbf{p}) = &-\frac{g^2}{16\pi}\Delta m m\left[\coth\left(\frac{\Delta m}{2T}\right) + 1\right]\\ &- \frac{g^2}{96\pi}\frac{\mathbf{p}^2}{2m}\left(\frac{\Delta m}{T}\right)^2\left[\frac{-3T + \Delta m \coth\left(\frac{\Delta m}{2T}\right)}{\sinh^2\left(\frac{\Delta m}{2T}\right)}\right].\end{aligned} \tag{19a}$$

while for the fundamental state the result is:

$$\begin{aligned}\text{Im}\,\Sigma_{\text{R}}(E_{\mathbf{p}}, \mathbf{p}) = &-\frac{g^2}{16\pi}\Delta m\, m\left[\coth\left(\frac{\Delta m}{2T}\right) - 1\right]\\ &-\frac{g^2}{96\pi}\frac{\mathbf{p}^2}{2m}\left(\frac{\Delta m}{T}\right)^2\left[\frac{-3T + \Delta m \coth\left(\frac{\Delta m}{2T}\right)}{\sinh^2\left(\frac{\Delta m}{2T}\right)}\right].\end{aligned} \tag{19b}$$

The corresponding decay rates, which correspond to the Feynman diagrams in Fig. 2, go as follows:

$$\Gamma_-^*(\mathbf{p}) = -\frac{1}{E_{\mathbf{p}}^*}\,\mathrm{Im}\,\Sigma_{\mathrm{R}}^*(E_{\mathbf{p}}^*, \mathbf{p}) = \frac{g^2}{8\pi}\Delta m\,[1+n(\Delta m)]\left(1-\frac{\mathbf{p}^2}{2m^2}\right)$$
$$+\frac{g^2}{24\pi}\Delta m\left(\frac{\mathbf{p}^2}{2m^2}\right)F(\Delta m/T) \tag{20a}$$
$$\Gamma_-(\mathbf{p}) = -\frac{1}{E_{\mathbf{p}}}\,\mathrm{Im}\,\Sigma_{\mathrm{R}}(E_{\mathbf{p}}, \mathbf{p}) = \frac{g^2}{8\pi}\Delta m\,n(\Delta m)\left(1-\frac{\mathbf{p}^2}{2m^2}\right)$$
$$+\frac{g^2}{24\pi}\Delta m\left(\frac{\mathbf{p}^2}{2m^2}\right)F(\Delta m/T), \tag{20b}$$

where $F(x)$ is a shorthand for

$$F(x) = \frac{-3x + x^2\coth(x/2)}{\sinh^2(x/2)}. \tag{21}$$

When the atom is at rest or the kinetic energy can be neglected in front of the rest energy, the previous equations reduce to:

$$\Gamma_-^* = \Gamma_-^*(\mathbf{0}) = \frac{g^2}{8\pi}\Delta m\,[1+n(\Delta m)] \tag{22a}$$

$$\Gamma_- = \Gamma_-(\mathbf{0}) = \frac{g^2}{8\pi}\Delta m\,n(\Delta m) \tag{22b}$$

The result obtained from the self-energy coincides with the one computed directly from the transition rates of Eqs. (14a) and (14b). One can also check that the vacuum results in the zero temperature limit are recovered.

It is important to point out that Eqs. (22a) and (22b) admit a double interpretation. On the one hand, from the point of view of field theory, they represent the decay rates of the particle excitations in a thermal bath. We would like to emphasize that in this context "decay" simply means going to a different state and not necessarily to a lower mass state. Notice also that the interaction preserves the total number of particle excitations, so that ϕ_M decays into ϕ_m and vice versa. On the other hand, from the point of view of the internal atomic states, Γ_- describes the probability of excitation of the fundamental state in a thermal bath and Γ_-^* describes the probability of (stimulated) decay of the excited state into the fundamental. For this reason, Γ_- and Γ_-^* also verify the 'atomic' detailed equilibrium condition $\Gamma_-^* = e^{-\Delta m/T}\,\Gamma_-$, wherein only the mass gap enters.

Finally, let us simply mention that at one loop there is no thermal contribution to the self-energy of the massless field Π because of the hypothesis that the massive fields ϕ_m and ϕ_M, which are interacting with χ, remain thermally unexcited.

5. GREEN FUNCTIONS IN A ONE-PARTICLE BACKGROUND

In this section we consider departure from equilibrium situation: we will compute the Green functions in a background state characterized by the presence of an atom in its fundamental state, on top of a thermal radiation bath. Since studying the propagators in this background involves creating an additional test particle, in total there will be two massive particles interacting with the thermal radiation.

The CTP formalism, which we have briefly described at the beginning of the previous section, still applies in this situation. In particular, recall that the self-energy component *ab* corresponds to the sum of all one-particle irreducible diagrams linking an *a* and a *b* vertex. Notice also that (8), which connects the retarded propagator and self-energy, does not depend on any specific property of thermal field theory but is valid in general. Therefore, in order to obtain the retarded propagator in this background, we will first compute the relevant self-energy components, then we will compute the retarded self-energy via the relation $\Sigma_{\rm R}(x, x') = \Sigma^{11}(x, x') + \Sigma^{12}(x, x')$, and in the end we will use (8) to connect the retarded self-energy to the retarded propagator. To this end, we also need the free propagators in the new background state, see the Appendix. The additional particle give rises to an additional on-shell term in the expression of the free propagators.

In a realistic situation, the atom would be distributed according to a certain density matrix $\rho(\mathbf{x}, \mathbf{x}')$ or equivalently to its corresponding Wigner function $W(\mathbf{p}, \mathbf{x})$, and the problem would not be translation-invariant. However, for simplicity, we shall assume that the atom has a well-defined momentum $\mathbf{p}'$. This implies that it is completely delocalized in position space and hence spatial translation invariance is preserved.

For computational simplicity, we will further assume that time translation invariance is also preserved, although this is not strictly correct because the background state is no longer stationary (the extra atom on top of the bath will thermalize[7]). Anyway, we expect that the general picture shown in this section is not substantially modified by this hypothesis and in the next section we will argue that the one-loop results are probably not affected at all.

Adopting this hypothesis, the Fourier transform of the retarded propagator can be isolated from Eq. (8):

$$G_{\rm R}(p; \mathbf{p}') = \frac{-i}{p^2 + m^2 + \Sigma_{\rm R}(p; \mathbf{p}')}. \tag{23}$$

[7] Moreover, we could have considered the case in which the added particle has reached equilibrium with the heat bath. In this case, the background state would have been stationary. This equilibrium situation is related to the present case in the following manner. There is an extra thermal averaging in self-energies which arises from the distribution of the added particle. This is presently under study.

By $G_{\mathrm{R}}(p;\mathbf{p}')$ we denote the retarded propagator in the state characterized by the extra on shell particle with momentum $\mathbf{p}'$.

One question we want investigate is whether in this background the imaginary part of the self-energy still admits an interpretation as a decay rate or a rate of approach to the equilibrium. Following the same kind of heuristic arguments that led to Eq. (11), the rate of approach to equilibrium of an ensemble of atoms is described by

$$\frac{\partial f(\mathbf{p},t)}{\partial t} = -\Gamma_-(\mathbf{p},[f^*])f(\mathbf{p},t) + \Gamma_-(\mathbf{p},[f^*])[1+f(\mathbf{p},t)], \tag{24a}$$

$$\frac{\partial f^*(\mathbf{p},t)}{\partial t} = -\Gamma^*_-(\mathbf{p},[f])f^*(\mathbf{p},t) + \Gamma^*_-(\mathbf{p},[f])[1+f^*(\mathbf{p},t)] \tag{24b}$$

The functions $f(\mathbf{p},t)$ and $f^*(\mathbf{p},t)$ are the distribution functions of the atoms in its fundamental and excited atomic states, respectively. For the case of a single particle, the distribution function $f(\mathbf{p},t)$ is connected to the density matrix of the state through

$$f(\mathbf{p},t) = N\int d^3\mathbf{x}\, d^3\mathbf{x}'\; e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}\,\rho(\mathbf{x},\mathbf{x}';t), \tag{25}$$

or, equivalently, to the Wigner function through

$$f(\mathbf{p},t) = N\int d^3\mathbf{x}\, W(\mathbf{x},\mathbf{p};t) \tag{26}$$

where N is a normalization constant and similarly for $f^*(\mathbf{p},t)$. The decay rate $\Gamma_-(\mathbf{p},[f^*])$ is the probability per unit time for an incoming light state of momentum $\mathbf{p}$ to decay to any other state on top of a background characterized by a distribution function of heavy states $f^*(\mathbf{p},t)$. Similarly, the creation rate $\Gamma_+(\mathbf{p},[f^*])$ is the probability per unit time for a particle with momentum $\mathbf{p}$ to be spontaneously created from the distribution of heavy particles characterized by $f^*(\mathbf{p},t)$. Analogous definitions apply for $\Gamma^*_-(\mathbf{p},[f])$ and $\Gamma^*_+(\mathbf{p},[f])$. In terms of decay amplitudes, these are expressed as

$$\Gamma^*_-(\mathbf{p}) = \frac{1}{2E^*_{\mathbf{p}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1+n(|\mathbf{q}|)}{2|\mathbf{q}|}\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{[1+f(\mathbf{k})]}{2E_{\mathbf{k}}}|\mathcal{M}^*_-|^2$$
$$\times(2\pi)^4\delta^{(3)}(\mathbf{p}-\mathbf{q}-\mathbf{k})\delta(E^*_{\mathbf{p}}-|\mathbf{q}|-E_{\mathbf{k}}), \tag{27a}$$

$$\Gamma_-(\mathbf{p}) = \frac{1}{2E_{\mathbf{p}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{n(|\mathbf{q}|)}{2|\mathbf{q}|}\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{[1+f^*(\mathbf{k})]}{2E^*_{\mathbf{k}}}|\mathcal{M}_-|^2$$
$$\times(2\pi)^4\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{k})\delta(E_{\mathbf{p}}+|\mathbf{q}|-E^*_{\mathbf{k}}), \tag{27b}$$

$$\Gamma^*_+(\mathbf{p}) = \frac{1}{2E^*_\mathbf{p}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{n(|\mathbf{q}|)}{2|\mathbf{q}|} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f(\mathbf{k})}{2E_\mathbf{k}} |\mathcal{M}^*_+|^2$$
$$\times (2\pi)^4 \delta^{(3)}(\mathbf{p} - \mathbf{q} - \mathbf{k}) \delta(E^*_\mathbf{p} - |\mathbf{q}| - E_\mathbf{k}), \quad (27c)$$

$$\Gamma_+(\mathbf{p}) = \frac{1}{2E_\mathbf{p}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1 + n(|\mathbf{q}|)}{2|\mathbf{q}|} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f^*(\mathbf{k})}{2E^*_\mathbf{k}} |\mathcal{M}_+|^2$$
$$\times (2\pi)^4 \delta^{(3)}(\mathbf{p} + \mathbf{q} - \mathbf{k}) \delta(E_\mathbf{p} + |\mathbf{q}| - E^*_\mathbf{k}). \quad (27d)$$

For simplicity, we have considered the radiation field to remain in thermal equilibrium but the expressions could be easily generalized in order to allow for an arbitrary distribution function of the radiation, $f'(\mathbf{q}, t)$, instead of the thermal distribution governed by the Bose–Einstein distribution function $n(|\mathbf{q}|)$. Finally, let us note that in our particular case $f(\mathbf{p}, 0) \propto \delta(\mathbf{p} - \mathbf{p}')$ and $f^*(\mathbf{p}, 0) = 0$.

In the beginning of this section we have used the notation G and Σ, but the relations also apply to the propagator and self-energy of the excited state, G^* and Σ^*, and those of the radiation field, Δ and Π:

$$G^*_\mathrm{R}(p; \mathbf{p}') = \frac{-i}{p^2 + M^2 + \Sigma^*_\mathrm{R}(p; \mathbf{p}')}, \qquad \Delta^*_\mathrm{R}(p; \mathbf{p}') = \frac{-i}{p^2 + \Pi^*_\mathrm{R}(q; \mathbf{p}')} \quad (28)$$

In the sequel, we will compute the modification of the self-energies of the radiation field χ and the massive field ϕ_M. At one loop the low-mass state ϕ_m will not receive modifications.

5.1. Radiation Field

For the radiation field we will focus on the imaginary part of the retarded self-energy. We can use for instance the following relation (see e.g. Arteaga *et al.*, 2004a):

$$\mathrm{Im}\,\Pi_\mathrm{R}(q; \mathbf{p}') = \frac{i}{2}[\Pi^{21}(q; \mathbf{p}') - \Pi^{12}(q; \mathbf{p}')] \quad (29)$$

The self-energy component Π^{12} is given by:

$$\Pi^{12}(q; \mathbf{p}') = -ig^2 m^2 \int \frac{d^4k}{(2\pi)^4} G^{(0)}_{12}(k; \mathbf{p}') G^{(0)*}_{12}(q - k), \quad (30)$$

where $G^{(0)*}_{12}(p)$ corresponds to the free vacuum propagator of ϕ_M and $G^{(0)}_{12}(k; \mathbf{p}')$ corresponds to the free propagator of ϕ_m in the background state $|\mathbf{p}'\rangle$ (see the Appendix). Restricting for the moment to the on-shell value and discarding the purely vacuum contribution (because the vacuum contribution corresponds to the decay rate of a massless particle in the vacuum, and we know that this process cannot happen on-shell), we get:

$$\Pi^{12}(|\mathbf{q}|,\mathbf{q};\mathbf{p}') = ig^2m^2\int \frac{d^4k}{(2\pi)^4}\frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{k}}}\delta(q^0-k^0+E^*_{\mathbf{q}-\mathbf{k}})\frac{(2\pi)^4}{2E_{\mathbf{k}}V}$$
$$\times\left[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\delta(k^0-E_{\mathbf{k}})+\delta^{(3)}(\mathbf{k}+\mathbf{p}')\delta(k^0+E_{\mathbf{k}})\right]. \quad (31)$$

Similarly, for Π^{21}:

$$\Pi^{21}(|\mathbf{q}|,\mathbf{q};\mathbf{p}') = ig^2m^2\int \frac{d^4k}{(2\pi)^4}\frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{k}}}\delta(q^0-k^0-E^*_{\mathbf{q}-\mathbf{k}})\frac{(2\pi)^4}{2E_{\mathbf{k}}V}$$
$$\times\left[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\delta(k^0-E_{\mathbf{k}})+\delta^{(3)}(\mathbf{k}+\mathbf{p}')\delta(k^0+E_{\mathbf{k}})\right]. \quad (32)$$

Summing the two previous results and developing the integrals with the delta functions one finds:

$$\begin{aligned}\mathrm{Im}\,\Pi_{\mathrm{R}}(|\mathbf{q}|,\mathbf{q};\mathbf{p}') = -\frac{g^2m^2}{4E_{\mathbf{p}'}V}\Bigg[&\frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{p}'}}\delta(|\mathbf{q}|-E_{\mathbf{p}'}-E^*_{\mathbf{q}-\mathbf{p}'})\\ &-\frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{p}'}}\delta(|\mathbf{q}|-E_{\mathbf{p}'}+E^*_{\mathbf{q}-\mathbf{p}'})\\ &+\frac{2\pi}{2E^*_{\mathbf{q}+\mathbf{p}}}\delta(|\mathbf{q}|+E_{\mathbf{p}'}-E^*_{\mathbf{q}-\mathbf{p}'})\\ &-\frac{2\pi}{2E^*_{\mathbf{q}+\mathbf{p}}}\delta(|\mathbf{q}|+E_{\mathbf{p}'}+E^*_{\mathbf{q}+\mathbf{p}})\Bigg]. \end{aligned} \quad (33)$$

From the four deltas, only one can contribute as:

$$\mathrm{Im}\,\Pi_{\mathrm{R}}(|\mathbf{q}|,\mathbf{q};\mathbf{p}') = -\frac{g^2m^2}{8VE^*_{\mathbf{q}+\mathbf{p}}E_{\mathbf{p}'}}2\pi\delta(|\mathbf{q}|+E_{\mathbf{p}'}-E^*_{\mathbf{q}+\mathbf{p}}). \quad (34)$$

One can assign a decay rate to this self-energy,

$$\begin{aligned}\gamma_-(|\mathbf{q}|,\mathbf{q};\mathbf{p}') &= -\frac{1}{|\mathbf{q}|}\,\mathrm{Im}\,\Pi_{\mathrm{R}}(|\mathbf{q}|,\mathbf{q};\mathbf{p}')\\ &= \frac{g^2m^2}{8V|\mathbf{q}|E^*_{\mathbf{q}+\mathbf{p}}E_{\mathbf{p}'}}2\pi\delta(|\mathbf{q}|+E_{\mathbf{p}'}-E^*_{\mathbf{q}+\mathbf{p}}). \end{aligned} \quad (35)$$

This decay rate, in fact, corresponds to the absorption rate of the radiation field by the excited atom in the background. The appearance of the spacetime volume V follows from the hypothesis that the atom is delocalized. In a more physical situation it would be replaced by the characteristic volume of the wavefunction of the atom or by the inverse density of particles per unit volume. In the case in

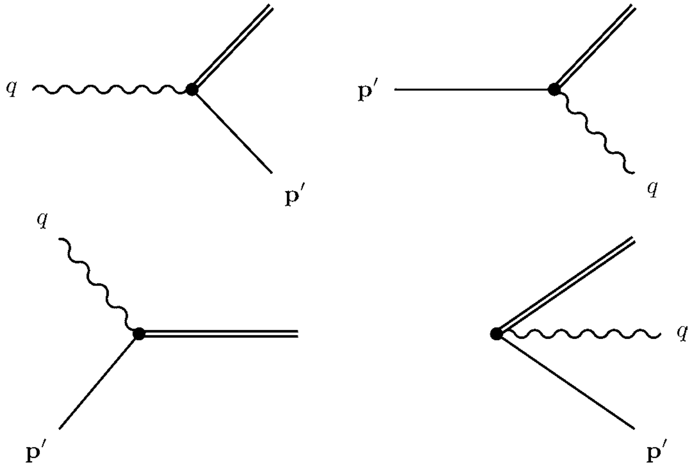

Fig. 3. Diagrams contributing to the off-shell imaginary part of the self-energy of the radiation field in a background characterized by the presence of an atom in its fundamental state. The two diagrams on the left show the contribution in form of decay rate and the diagrams on the right the contribution as creation rates. The only diagram contributing on shell, $q = (|\mathbf{q}|, \mathbf{q})$, is the one on the bottom on the left.

which the atom is originally at rest, we get:

$$\gamma_-(|\mathbf{q}|, \mathbf{q}; \mathbf{p}') \approx \frac{g^2}{8V|\mathbf{q}|} 2\pi \delta(|\mathbf{q}| - \Delta m). \tag{36}$$

We can check that this self-energy truly corresponds to a decay rate computing directly the relevant Feynman diagram, shown in Fig. 3, by using expressions analogous to those of Eqs. (27a)–(27d).

In Eq. (33), only one of the four deltas contributed on-shell. We can go somewhat further by reexpressing this equation off the photon mass-shell:

$$\begin{aligned}\Delta \mathrm{Im}\, \Pi_{\mathrm{R}}(q; \mathbf{p}') = -\frac{g^2 m^2}{4E_{\mathbf{p}'} V} \Bigg[& \frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{p}'}} \delta(q^0 - E_{\mathbf{p}'} - E^*_{\mathbf{q}-\mathbf{p}'}) \\ & - \frac{2\pi}{2E^*_{\mathbf{q}-\mathbf{p}'}} \delta(q^0 - E_{\mathbf{p}'} + E^*_{\mathbf{q}-\mathbf{p}'}) \\ & + \frac{2\pi}{2E^*_{\mathbf{q}+\mathbf{p}}} \delta(q^0 + E_{\mathbf{p}'} - E^*_{\mathbf{q}+\mathbf{p}}) \\ & - \frac{2\pi}{2E^*_{\mathbf{q}+\mathbf{p}}} \delta(q^0 + E_{\mathbf{p}'} + E^*_{\mathbf{q}+\mathbf{p}}) \Bigg],\end{aligned} \tag{37}$$

where $\Delta\text{Im}\,\Pi_{\text{R}}(q;\mathbf{p}')$ is the correction to the equilibrium off-shell self-energy. The first and third terms on the right hand side of this equation give the modification of the decay rate due to the background particle with momentum $\mathbf{p}'$, while the second and fourth terms give the modification of the creation rate, taking into account the Bose–Einstein statistics of the particles, as it can be seen in Fig. 3. So, the imaginary part of the retarded self-energy is proportional to the decay rate minus the creation rate,

$$\text{Im}\,\Pi_{\text{R}}(q;\mathbf{p}') = -\frac{1}{q^0}\left[\gamma_-(q;\mathbf{p}') - \gamma_+(q;\mathbf{p}')\right], \tag{38}$$

thereby recovering the same relation of Eq. (10) that applied to massive fields. Compare also the previous equation with the corresponding expression in the vacuum, Eq. (6).

5.2. Massive Field ϕ_M

In this case we start by the computation of the real part of the self-energy. Using the property $\text{Re}\,\Sigma^*_{\text{R}}(p;\mathbf{p}') = \text{Re}\,\Sigma^{11*}(p;\mathbf{p}')$ (see e.g. Arteaga *et al.*, 2004b) and evaluating non-equilibrium contribution to the second Feynman diagram in Fig. 1, we get

$$\begin{aligned}\Delta\text{Re}\,\Sigma^*_{\text{R}}(p;\mathbf{p}') = -m^2g^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{-(p^0-k^0)^2+(\mathbf{p}+\mathbf{k})^2}\\ \times\frac{(2\pi)^4}{2E_{\mathbf{p}'}V}\left[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\delta(k^0-E_{\mathbf{p}'}) + \delta^{(3)}(\mathbf{k}+\mathbf{p}')\delta(k^0-E_{\mathbf{p}'})\right],\end{aligned} \tag{39}$$

which gives

$$\Delta\text{Re}\,\Sigma^*_{\text{R}}(p;\mathbf{p}') = \frac{m^2g^2}{2E_{\mathbf{p}'}}\left(\frac{1}{(\mathbf{p}-\mathbf{p}')^2-(p^0-E_{\mathbf{k}'})^2} + \frac{1}{(\mathbf{p}+\mathbf{p}')^2-(p^0+E_{\mathbf{k}'})^2}\right). \tag{40}$$

Evaluated in the mass-shell and in the approximation of slowly moving heavy atoms, one has:

$$\Delta\text{Re}\,\Sigma^*_{\text{R}}(E^*_{\mathbf{p}},\mathbf{p};\mathbf{p}') = \frac{g^2}{2}\frac{m/V}{(\mathbf{p}-\mathbf{p}')^2-(\Delta m)^2}. \tag{41}$$

Notice that the previous expression cannot be absorbed in the mass term.

For the case of the imaginary part of the self-energy, we can repeat a similar calculation to the previous section. Now we have to take into account not only that the light field is in a one-particle state but also that the massless field is in a thermal state. Following similar same steps as in the previous case, we get:

$$\begin{aligned}\Delta\mathrm{Im}\,\Sigma^*_{\mathrm{R}}(p;\mathbf{p}') = -\frac{g^2m^2}{4E_{\mathbf{p}'}V}\Bigg[&\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(p^0-E_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|)[1+n(|\mathbf{p}-\mathbf{p}'|)]\\
&-\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(p^0-E_{\mathbf{p}'}+|\mathbf{p}-\mathbf{p}'|)[1+n(|\mathbf{p}-\mathbf{p}'|)]\\
&+\frac{2\pi}{2|\mathbf{p}+\mathbf{p}'|}\delta(p^0+E_{\mathbf{p}'}-|\mathbf{p}+\mathbf{p}'|)[1+n(|\mathbf{p}+\mathbf{p}'|)]\\
&-\frac{2\pi}{2|\mathbf{p}+\mathbf{p}'|}\delta(p^0+E_{\mathbf{p}'}+|\mathbf{p}+\mathbf{p}'|)[1+n(|\mathbf{p}+\mathbf{p}'|)]\\
&+\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(p^0-E_{\mathbf{p}'}+|\mathbf{p}-\mathbf{p}'|)n(|\mathbf{p}-\mathbf{p}'|)\\
&-\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(p^0-E_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|)n(|\mathbf{p}-\mathbf{p}'|)\\
&+\frac{2\pi}{2|\mathbf{p}+\mathbf{p}'|}\delta(p^0+E_{\mathbf{p}'}+|\mathbf{p}+\mathbf{p}'|)n(|\mathbf{p}+\mathbf{p}'|)\\
&-\frac{2\pi}{2|\mathbf{p}+\mathbf{p}'|}\delta(p^0+E_{\mathbf{p}'}-|\mathbf{p}+\mathbf{p}'|)n(|\mathbf{p}+\mathbf{p}'|)\Bigg]. \qquad (42)\end{aligned}$$

These eight terms can be similarly interpreted as the modification to the decay and creation rates of the massive state due to the presence of the extra atom and the thermal massless particle. The terms with a global negative sign correspond to decay and the ones with a global positive sign to creation. So again we have

$$\Delta\mathrm{Im}\,\Sigma^*_{\mathrm{R}}(p;\mathbf{p}') = -\frac{1}{p^0}\left[\Gamma^*_-(p;\mathbf{p}')-\Gamma^*_+(p;\mathbf{p}')\right]. \qquad (43)$$

On-shell only the first and sixth terms contribute as

$$\begin{aligned}\Delta\mathrm{Im}\,\Sigma^*_{\mathrm{R}}(E_{\mathbf{p}},\mathbf{p};\mathbf{p}') = -\frac{g^2m^2}{4E_{\mathbf{p}'}V}\Bigg[&\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'}\\
&-|\mathbf{p}-\mathbf{p}'|)[1+n(|\mathbf{p}-\mathbf{p}'|)]\\
&-\frac{2\pi}{2|\mathbf{p}-\mathbf{p}'|}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|)n(|\mathbf{p}-\mathbf{p}'|)\Bigg]. \qquad (44)\end{aligned}$$

The two Feynman diagrams corresponding to these two terms are shown in Fig. 4, the first being a decay term and the second a creation term. The two terms can be

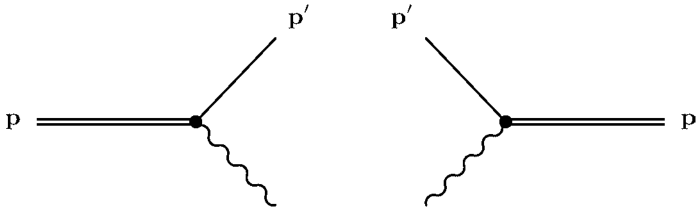

Fig. 4. Diagrams contributing to the off-shell imaginary part of the self-energy of an excited atom, with momentum **p**, in a background characterized by the presence of another atom in its fundamental state, with momentum **p**′. The diagram on the left corresponds to the decay of the excited atom and one on the right corresponds to the creation.

added to give a net rate of approach to equilibrium:

$$\Delta\Gamma^*(\mathbf{p};\mathbf{p}') = \frac{g^2 m^2}{8V|\mathbf{p}-\mathbf{p}'|E^*_{\mathbf{p}}E_{\mathbf{p}'}} 2\pi\delta(E^*_{\mathbf{p}} - E_{\mathbf{p}'} - |\mathbf{p}-\mathbf{p}'|). \tag{45}$$

Notice that the thermal contributions cancel exactly. In the case in which the atom is at rest,

$$\Delta\Gamma^* = \frac{g^2}{8V\Delta m} 2\pi\delta(\Delta m - |\mathbf{p}-\mathbf{p}'|). \tag{46}$$

We end this section by considering the fact that the interaction preserves the total atomic number, i.e.,

$$\frac{d}{dt}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[f(\mathbf{p},t) + f^*(\mathbf{p},t)\right] = 0. \tag{47}$$

According to Eqs. (27a)–(27d), a sufficient condition for this to hold is that

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}[1 + f^*(\mathbf{p},t)]\Gamma^*_+(\mathbf{p}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\mathbf{p},t)\Gamma_-(\mathbf{p}), \tag{48a}$$

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}[1 + f(\mathbf{p},t)]\Gamma_+(\mathbf{p}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f^*(\mathbf{p},t)\Gamma^*_-(\mathbf{p}), \tag{48b}$$

which expresses that the heavy states decay into the light ones and vice versa. Evaluating these equations at initial time and taking into account that $f^*(\mathbf{p},0) = 0$, we have

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}\Gamma^*_+(\mathbf{p}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\mathbf{p})\Gamma_-(\mathbf{p}), \tag{49a}$$

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3}[1 + f(\mathbf{p})]\Gamma_+(\mathbf{p}) = 0. \tag{49b}$$

The second equation is consistent with the fact that the creation rate of light particles vanishes, $\Gamma_+(\mathbf{p}) = 0$. Notice that the decay rate of light particles $\Gamma_-(\mathbf{p})$ is just the thermal value given by Eq. (20b). Notice also that the creation rate of the heavy states comes entirely from the non-equilibrium part and is given by $\Gamma_+^*(\mathbf{p}) = \Delta\Gamma^*(\mathbf{p})n(|\mathbf{p} - \mathbf{p}'|)$. One can therefore check that effectively Eq. (49a) holds. Hence, this provides a test of the consistency of the theory.

6. SUMMARY AND DISCUSSION

We have studied a field theory description of a relativistic two-level atom in a radiation field. By means of the analysis of the poles of the retarded propagators of the two fields ϕ_m and ϕ_M we have recovered the basic properties of the atomic levels: At zero temperature, the fundamental state of the atom is stable, while the excited state has a finite lifetime inversely proportional to the mass gap between the two states Δm. At finite temperature, the fundamental state is no longer stable but has an characteristic excitation time given by Γ_-^{-1}, which is proportional to the density of radiation. The decay rate of the excited state is enhanced because of the Bose–Einstein statistics.

These results are of course perfectly known, but it is interesting to see how the atomic description emerges from the field-theoretic language. For instance, from the point of view of field theory, Γ_- and Γ_-^* are interpreted as the decay rates of the particle excitations of the fields ϕ_m and ϕ_M, respectively. From the point of view of the internal atomic states, Γ_- corresponds to the probability of transition from the fundamental to the excited state per unit time and Γ_-^* corresponds to the probability of transition from the exited to the fundamental state. Since the atom has only two levels, Γ_- and Γ_-^* obey a detailed equilibrium condition. Notice that the field creation rates Γ_+ and Γ_+^* vanish since no atoms can be spontaneously created from a thermal bath.

We have also analysed the same system in a non-equilibrium background consisting of an extra atom on top of the thermal bath. The results show that the usual interpretation of the imaginary part of the self-energy as a difference of destruction and creation rates holds in this out-of-equilibrium situation both for the radiation field and the massive field.

In the case of a massless particle, its self-energy acquires an imaginary part, which reflects the fact that radiation can be absorbed by the atom in the background. This is in contrast to the vacuum, where massless particles are always stable and, thus, do not develop an imaginary part in the poles of the propagators. In the case of the an exited atom, its effective decay rate increases because of the presence of the additional atom. The contribution to the effective rate can be split as the difference of two terms: first, a net decay rate minus a creation rate; and second, the reason why the atom in the fundamental state can become exited. It is interesting to note that the increase of the effective decay rate is independent of the temperature, although each term separately depends on it. This is a consequence of the fact that the system effectively behaves as linear (in linear systems the retarded propagator is independent of the background state).

However, the aforementioned results are preliminary because we have not properly taken into account the fact that the background is not stationary. To get manageable expressions, we have assumed that the Green functions are time translation invariant, which cannot be the case if the background is thermalizing. It is likely that our one-loop results are not affected because the effects of the background thermalization on the dynamics of the test particle are of subleading order in the coupling constant g. Moreover, in the case in which the background atom is at rest, the thermalization effects will basically consist in the fact that the internal excited state will become partially populated. We could thus recover the equilibrium situation by considering that the background atom is not in the fundamental but in a thermally averaged internal state, which amounts to an additional thermal average of our expressions. Therefore, we do not expect that the qualitative description in the non-equilibrium background is significantly affected by this additional hypothesis.

We have also studied the equations governing the time evolution of the distribution function of the two atomic states, f and f^*, as a function of the decay and creation rates, both in a standard near-equilibrium situation [Eq. (11)] and in an out-of-equilibrium situation [Eqs. (24a) and (24b)]. It should be noted that the non-equilibrium expression is in fact a direct generalization of the equilibrium equation: it is enough to reinterpret every term of the thermal expression into the more general non-equilibrium situation.

Equations (11), and (24a) and (24b) show that the imaginary part of the self-energy gives, at least, the time evolution of the distribution functions, which roughly speaking amounts to the squared modulus of the wavefunction of the incoming test particles. These equations are derived from arguments of plausibility; however, one should be able to deduce them from the equations governing the evolution of the density matrix of the atomic states in the radiation field, at least under some reasonable approximation. We should note that, while the basic features of the atomic behaviour can be reproduced from the analysis of the poles of the field propagators, the fine details of the evolution of the density matrix cannot probably be obtained from a field theoretic pole analysis.

Although most of the discussion given earlier in the paper has been related to quantities extracted from the imaginary part of the pole, we have also briefly analysed the real part of the poles, which give the modification of the dispersion relation. The finite temperature contribution to the dispersion relation of the atomic states can be absorbed in a thermal contribution to the mass, although this is not what one naively expects. This also happens in other situations (Arteaga *et al.*, 2004b; Donoghue *et al.*, 1985). However, in out-of-equilibrium situations there are effectively contributions to the dispersion relation that cannot be absorbed in the thermal mass. This points towards the direction that the absorption of the modified dispersion relation in the mass is a property linked to the particular features of thermal field theory.

APPENDIX: FREE PROPAGATORS IN A ONE-PARTICLE STATE BACKGROUND

In this appendix we compute the free propagators for the field ϕ_m in the background state characterized by the presence of a one-particle excitation with momentum $\mathbf{p}'$. This state will be denoted by $|\mathbf{p}'\rangle$. We need the full set of CTP propagators, i.e.,

$$G_{ab}(x, x'; \mathbf{p}') = \begin{pmatrix} \langle \mathbf{p}'|T\hat{\phi}_m(x)\hat{\phi}_m(x')|\mathbf{p}'\rangle & \langle \mathbf{p}'|\hat{\phi}_m(x)\hat{\phi}_m(x')|\mathbf{p}'\rangle \\ \langle \mathbf{p}'|\hat{\phi}_m(x')\hat{\phi}_m(x')|\mathbf{p}'\rangle & \langle \mathbf{p}'|\widetilde{T}\hat{\phi}_m(x)\hat{\phi}_m(x')|\mathbf{p}'\rangle. \end{pmatrix} \tag{A.1}$$

From now on we drop the subindex m to simplify the notation.

In terms of creation operators, the state $|\mathbf{p}'\rangle$ corresponds to

$$|\mathbf{p}'\rangle = \frac{(2\pi)^{3/2}}{\sqrt{2E_{\mathbf{p}'}V}}\hat{a}^\dagger_{\mathbf{p}'}|0\rangle, \tag{A.2}$$

where V is the volume of the space. Let us check that the normalization is the correct one:

$$|||\mathbf{p}'\rangle||^2 = |\langle\mathbf{p}'|\mathbf{p}'\rangle|^2\frac{(2\pi)^3}{2E_{\mathbf{p}'}V}\langle 0|a_{\mathbf{p}}a^\dagger_{\mathbf{p}'}|0\rangle = \frac{(2\pi)^3}{2E_{\mathbf{p}'}V} = \frac{(2\pi)^3}{V}\delta^{(3)}(0),$$

which is equal to one with a suitable regularization of the delta function (for instance working in a cubic box of length $V^{1/3}$). We are using a convention in which

$$[\hat{a}_{\mathbf{p}'}, \hat{a}^\dagger_{\mathbf{q}}] = 2E_{\mathbf{p}'}\delta^{(3)}(\mathbf{p}' - \mathbf{q}). \tag{A.3}$$

Let us now compute a Whightman function in the background state $|\mathbf{p}'\rangle$:

$$\begin{aligned} G^{(0)}_{21}(t, \mathbf{x}; t', \mathbf{x}'; \mathbf{p}') &= \langle\mathbf{p}'|\hat{\phi}(t, x)\hat{\phi}(t', x')|\mathbf{p}'\rangle = \frac{(2\pi)^3}{2E_{\mathbf{p}'}V}\langle 0|\hat{a}_{\mathbf{p}'}\hat{\phi}(t, x)\hat{\phi}(t', x')\hat{a}^\dagger_{\mathbf{p}'}|0\rangle \\ &= \frac{(2\pi)^3}{2E_{\mathbf{p}'}V}\int\frac{d^3\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3}\frac{d^3\mathbf{k}'}{2E_{\mathbf{k}'}(2\pi)^3} \\ &\quad\times e^{-iE_{\mathbf{p}}t+i\mathbf{p}\cdot\mathbf{x}}\,e^{iE_{\mathbf{k}'}t'-i\mathbf{k}'\cdot\mathbf{x}'}\langle 0|\hat{a}_{\mathbf{p}'}\hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}'}\hat{a}^\dagger_{\mathbf{p}'}|0\rangle \\ &\quad+\frac{(2\pi)^3}{2E_{\mathbf{p}'}V}\int\frac{d^3\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3}\frac{d^3\mathbf{k}'}{2E_{\mathbf{k}'}(2\pi)^3} \\ &\quad\times e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}}\,e^{-iE_{\mathbf{k}'}t'+i\mathbf{k}'\cdot\mathbf{x}'}\langle 0|\hat{a}_{\mathbf{p}'}\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}'}\hat{a}^\dagger_{\mathbf{p}'}|0\rangle, \end{aligned} \tag{A.4}$$

where we have used the expansion of the field operator in terms of creation and annihilation operators:

$$\hat{\phi}(t,x) = \int \frac{d^3\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3}\frac{1}{2E_{\mathbf{k}}}(\hat{a}_{\mathbf{k}}\, e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^{\dagger}_{\mathbf{k}}\, e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}}). \tag{A.5}$$

Next, we evaluate explicitly the different contractions of the creation and annihilation operators:

$$\begin{aligned} G^{(0)}_{21}(t,\mathbf{x};t',\mathbf{x}';\mathbf{p}') = \int \frac{d^3\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3}\, e^{-iE_{\mathbf{k}}(t-t')+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \\ +\frac{(2\pi)^3}{2E_{\mathbf{p}'}V}\left[e^{-iE_{\mathbf{k}}(t-t')+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} + e^{iE_{\mathbf{k}}(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\right], \end{aligned} \tag{A.6}$$

which in the Fourier representation becomes

$$\begin{aligned} G^{(0)}_{21}(k;\mathbf{p}') = \frac{2\pi}{2E_{\mathbf{k}}}\delta(k^0 - E_{\mathbf{k}}) + \frac{(2\pi)^4}{2E_{\mathbf{p}'}V}\big[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\delta(k^0 - E_{\mathbf{k}}) \\ +\delta^{(3)}(\mathbf{k}+\mathbf{p}')\delta(k^0 + E_{\mathbf{k}})\big]. \end{aligned} \tag{A.7}$$

Using the properties of the delta functions, the result can be further reorganized as

$$\begin{aligned} G^{(0)}_{21}(k;\mathbf{p}') = 2\pi\delta(k^2+m^2)\left[\theta(k^0) + \frac{(2\pi)^3}{V}\delta^{(3)}(\mathbf{k}-\mathbf{p}')\theta(k^0)\right. \\ \left. + \frac{(2\pi)^3}{V}\delta^{(3)}(\mathbf{k}+\mathbf{p}')\theta(-k^0)\right]. \end{aligned} \tag{A.8}$$

Repeating a similar calculation for the Feynman propagator we find:

$$\begin{aligned} G^{(0)}_{11}(k;\mathbf{p}') = \frac{-i}{p^2+m^2-i\epsilon} + 2\pi\delta(k^2+m^2)\frac{(2\pi)^3}{V} \\ \times\big[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\theta(k^0) + \delta^{(3)}(\mathbf{k}+\mathbf{p}')\theta(-k^0)\big]. \end{aligned} \tag{A.9}$$

The complete matrix of CTP propagators is given by

$$\begin{aligned} G^{(0)}_{ab}(k;\mathbf{p}') = \begin{pmatrix} \frac{-i}{k^2+m^2-i\epsilon} & 2\pi\delta(k^2+m^2)\theta(-k^0) \\ 2\pi\delta(p^2+m^2)\theta(k^0) & \frac{i}{k^2+m^2+i\epsilon} \end{pmatrix} \\ + \delta(k^2+m^2)\frac{(2\pi)^4}{V}\big[\delta^{(3)}(\mathbf{k}-\mathbf{p}')\theta(k^0) \\ +, \delta^{(3)}(\mathbf{k}+\mathbf{p}')\theta(-k^0)\big]\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \end{aligned} \tag{A.10}$$

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